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On Renormalization Group in Abstract QFT

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Abstract

The basics of RG equations for generic partition functions are briefly reviewed, keeping in mind an application to the Polyakov-de Boer-Verlindes description of the holomorphic RG flow.

1 RG versus (generalized) Laplace equations

The notion and properties of the (generalized) renormalization group (RG) [1, 2, 3] acquire a new attention last years, primarily because of the interest to its hidden (quasi)integrable sturcture (related to Whitham dynamics [4], see [5] and references therein) and to its implicit occurence in the phenomena like AdS/CFT correspondence [6] (these studies are focused on the so called holomorphic RG flows [7, 8, 9, 10]). The purpose of this note is to give a concise survey of the basics of abstract RG theory, separating generic features from the pecularities of particular models. One of our goals is to explicitly formulate the controversies between the different concepts, which people are now trying to unify. It is a resolution of these controversies, which should provide a real understanding.

The basic notion of modern quantum field/string theory is partition function (exponentiated effective action, the generating function for all the correlation functions in the theory in all possible vacua), resulting from functional integration over fields with an action, formed by a complete set of operators. Importantly, there are two different notions of completeness, see sect.2.2 below for the definitions. The exact RG *a la* J.Polchinski [2], possessing a formulation in terms of the diffeomorphisms in the moduli space $\mathcal{M}$ of theories [11], requires a *strong* completeness. It guarantees that the linear differential equations emerge for the partition function, and exponentiation of the corresponding vector field provides one-parametric families (flow lines) in *Diff* $\mathcal{M}$ describing the RG flow. At the same time, in most interesting examples only a *weak* completeness is assumed[*]: it is the one which is supposedly enough for integrability etc [12]. However, as natural for integrable systems, the weak completeness implies that partition function (interpreted as an element of some Hilbert space, see [11]) satisfies some differential equation in $\mathcal{M}$, which are rarely *linear* in time (coupling constant) derivatives. The typical example is the Casimir (generalized Laplace) equation for the zonal spherical functions, if of the second order it is an ordinary Laplace equation. The other avatars of the same equation are the W- (in particular Virasoro) constraints in matrix models [13, 14, 15, 16] and the Hamilton-Jacobi equations for the large-N Yang-Mills partition functions [9], studied in the context of the AdS/CFT correspondence [6, 7, 10]. Unfortunately, non-linear differential equations do not possess an RG-like interpretation, at least naively. They are rather associated with the huge group *Dop*$\mathcal{M}$ of all differential operators on $\mathcal{M}$, and there is no obvious way to associate them with the elements of a much smaller diffeomorphism group *Diff* $\mathcal{M}$. Perhaps surprisingly, an old result from the matrix models theory [17] implies that some intermediate notion can exist: despite matrix model provides only a weakly complete partition function, an explicit transformation of time-variables (coupling constants) is known, which (almost) eliminates the dependence on the size N of the matrix – what one would expect the exact RG to provide. A possible explanation of this phenomenon is that the *entire* set of the Ward identities (i.e. the full power of integrability) was used in the calculation of [17], not just a single Virasoro constraint L_2 (which is a counterpart of conventional Polchinski's equation).

In refs.[10] a seemingly artificial trick was suggested to resolve the controversy between the linear (in coupling-constants derivatives) notion of RG and the quadratic (at best) form of the Laplace/Virasoro/Hamilton-Jacobi equation. Namely, it was suggested to decompose the effective action into the contributions from over and from below the normalization point μ. The problem with this idea is that generically,

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[*]It goes without saying that in conventional field theories one rarely requires even a weak completeness. Then the notion of exact renormalization group is usually substituted by a renormalizability requirement, which concerns nothing but the singularities in the vicinities of the critical points, ultraviolet or infrared.

even for the *strongly* complete partition functions, these two contributions are of different nature [11]: the one from above μ can indeed be considered as effective action, while the one from below μ is rather a differential operator acting on functions on the moduli space of coupling constants. A special procedure is needed to extract a kind of a kernel from this operator, which can be interpreted as something similar to an effective action. This can be done in special approximations and with certain reservations. An obvious example when such decomposition exists, is provided by the quasiclassical approximation, when the shift of effective action with the changing μ is described by the tree (one-particle-reducible) diagrams, but it is not fully satisfactory: there are no such diagrams in the case of matrix models – which seems to be a prototype of the actually interesting situations. Despite quasiclassical approximation does not work in this simple way, it is known [17] that the relevant shift does occur in the matrix models, and is indeed similar in many respects [18] to the ansatz of [10], but no straightforward way is known to derive it "from the general principles" in the leading N^{-1} approximation. Worse than that, the very notion of normalization point becomes somewhat subtle for the *weakly* complete partition functions, of which the matrix models is an example.

In this note we do not suggest any definite conclusion from this description of problems.

As already mentioned, it can happen that the controversy is resolved if the full power of integrability (the full set of the Ward identities for the weakly complete partition function) is taken into account. Clarification of this option is intimately related to understanding the origins of a generalized AdS/CFT correspondence, i.e. of the representation of generic exact partition functions (not necessarily of a CFT) in terms of gravity theories (not necessarily on AdS). The full set of Ward identities for the weakly complete partition functions is implied by invariance of the integral (1) under arbitrary coordinate transformations in $\mathcal{A}$ (arbitrary changes of integration variables) [14]. In the strongly complete case, when the Ward identities are linear differential equations and each of them generates an RG flow, Polchinski's flow (10) being just one of them, the general covariance in the $\mathcal{A}$ space leads to the general covariance in the $\mathcal{M}$ space. This means that the partition function is essentially an invariant of general coordinate transformations in $\mathcal{M}$, thus giving a most natural object for a gravity theory on $\mathcal{M}$. This property is broken in realistic models by boundary conditions φ_0 and by the lack of the strong completeness: only the weak one is usually natural. Sometimes, the deviation from linearity in the Ward identities can be interpreted as "a quantum effect" in (1): the terms non-linear in t-derivatives come from the change of measure, not action, in (1) under reparametrizations of $\mathcal{A}$ [14], but this does not make one free of handling these non-linearities. It is very natural to assume that in the weakly complete case $Z(t)$ is still interpretable in terms of a gravity theory on $\mathcal{M}$, but is a slightly less trivial object than just an invariant. Within the AdS/CFT correspondence [6], this is rather a wave function, similarly to the considerations from the perspective of integrability theory [11]. In other words, $Z(t)$ is an invariant not of the *Diff* $\mathcal{M}$ subgroup of *Dop* $\mathcal{M}$ but of some other subgroup which should be a kind of a smooth deformation of *Diff* $\mathcal{M}$. The relation between the Feynman diagrams in field theory and the generators of *Dop* $\mathcal{M}$ [11] implied by considerations of [19] is one of the new tools to attack the problem.

Another, less attractive but simpler option is to sacrifice the *exact* RG (i.e. abandon the hope to find *some* substitute of Polchinski's equation, which indeed holds exactly in the weakly complete case and still has something to do with the diffeomorphism group *Diff* $\mathcal{M}$), and try to resolve the controversy near the critical points, in neglect of non-singular contributions. Then the power of the RG methods will be strongly reduced (just to the level they have in conventional quantum field theory), and even if it can help, this is hardly a satisfactory result. A slightly more interesting version of this option is that the *linear* RG is an effective object, valid for description of effective actions near the critical points, but different near different points. When extrapolated from the vicinity of one point to another, RG dynamics becomes non-linear, and linear RG equations are nothing but an approximation to non-linear Laplace/Virasoro/Hanilton-Jacobi equations (which generically are not even quadratic). This option, familiar from the studies of matrix models [14, 16, 17] can be the closest in spirit to the suggestion of refs.[10].

The rest of this paper contains just a brief comment on the terminology, used in above considerations.

2 Partition functions

The partition function

$$Z(G;\varphi_0;t) = \int_{\mathcal{A};\varphi_0} D\phi \exp\left(-\frac{1}{2}\phi G\phi + A(t;\phi)\right) \equiv \langle\langle 1 \rangle\rangle \tag{1}$$

depends on:

- the background fields φ_0;
- the coupling constants t;
- the metric G.

2.1 The fields

In eq.(1) $\mathcal{A}$ denotes the space of quantum fields (domain of integration in the functional integral). In D-dimensional field theory it is a D-loop space of maps from the D-dimensional "world sheet" (space-time) W into a target space T: $\mathcal{A} = \{\text{maps } W \to T\} = L^D_W(T)$. In the (second quantized) string field theory W is itself a space of loops in the space-time (while in the first-quantized theories the space-time plays instead the role of the target space T). When W is not compact, one needs to impose the boundary conditions at its boundary: $\varphi_0 \in \{\text{maps } \partial W \to T\}$. In most cases partition functions are non-vanishing only when the boundary conditions belong to some (co)homologies of the target space, $\varphi_0 \in H^*(T)$.

2.2 The coupling constants

The coupling constants parametrize the shape of the action

$$A(t,\phi) = \sum_{n\in B} t^{(n)} \mathcal{O}_n(\phi) \tag{2}$$

where the sum goes over some complete set B of functions $\mathcal{O}_n(\phi)$, not obligatory finite or even discrete. The space $\mathcal{M} \subset \mathrm{Fun}(\mathcal{A})$ of actions, parametrized by the coupling constants $t^{(n)}$, is refered to as the moduli space of theories. The actions usually take values in numbers or, more generally, in certain rings, perhaps, non-commutative. The space $\mathrm{Fun}(\mathcal{A})$ of all functions of ϕ is always a ring, but this need not be true about the moduli space $\mathcal{M}$, which could be as small a subset as one likes. However, the interesting notion of partition function arises only if the completeness requirement is imposed on $\mathcal{M}$ [12]. There are two different degrees of completeness, relevant for discussions of partition functions. In the first case (strong completeness) the functions $\mathcal{O}_n(\phi)$ form a *linear* basis in $\mathrm{Fun}(\mathcal{A})$, then $\mathcal{M}$ is essentially the same as $\mathrm{Fun}(\mathcal{A})$ itself. In the second case (weak completeness) the functions $\mathcal{O}_n$ generate $\mathrm{Fun}(\mathcal{A})$ as a ring, i.e. arbitrary function of ϕ can be decomposed into a sum of *multi*linear combination of $\mathcal{O}_n$'s. In the case of strong completeness the notion of RG is absolutely straightforward [2, 3, 11], but there is no clear idea how RG can be formulated in the case of weak completeness (which is more relevant for most modern considerations[†]).

2.3 The metric

In quantum field theory the metric G is needed at least for two purposes: to define perturbation theory as a formal sum over Feynman diagrams and to regularize the original functional integral. Regularization is needed whenever $\mathrm{Vol}\mathcal{A} = \int_{\mathcal{A}} D\phi = \infty$, then the factor $\exp\left(-\frac{1}{2}\phi G\phi\right)$ helps to make integrals finite (see below).

One often explicitly extracts from $A(\phi)$ not only the quadratic term $-\frac{1}{2}\phi G\phi$ with the metric, but also the linear source term $J\phi$ and the "vacuum energy" $A_0 = A(\phi = 0)$.

2.4 Normalization point

The Kadanoff-Wilson RG occurs when a filtration is defined in the space of fields, i.e. a map from positive numbers (real or integer, accordingly the RG is continuous or discrete) into the set $2^{\mathcal{A}}$ of the subsets of $\mathcal{A}$, such that

$$\mathcal{A}_\infty \subset \mathcal{A}_\lambda \subset \mathcal{A}_\mu \subset \mathcal{A}_0 = \mathcal{A}/H^*(T) \quad \forall \mu < \lambda \tag{3}$$

Accordingly the complements $\mathcal{B}_\mu = \mathcal{A}/\mathcal{A}_\mu$ of $\mathcal{A}_\mu$ in $\mathcal{A}$ satisfy

$$\mathcal{B}_0 \subset \mathcal{B}_\mu \subset \mathcal{B}_\lambda \subset \mathcal{B}_\infty = \mathcal{A} \quad \forall \mu < \lambda \tag{4}$$

One usually assumes that in the infra-red limit, $\mu = 0$, the space $\mathcal{B}_\mu$ shrinks to the space $\mathcal{B}_0 = H^*(T)$ of vacua, while in the ultraviolet limit, $\mu = \infty$, the space $\mathcal{B}_\infty$ coincides with the entire $\mathcal{A}$.

Given such a filtration, one can define a μ-dependent partition function

$$Z_\mu(G;\varphi_\mu;t) = \exp\left(-\frac{1}{2}\varphi_\mu G\varphi_\mu\right) \int_{\mathcal{A}_\mu;\varphi_\mu} D\phi \exp\left(-\frac{1}{2}\phi G\phi + A(t;\phi+\varphi_\mu)\right) \equiv \exp\left(-\frac{1}{2}\varphi_\mu G\varphi_\mu + A_\mu(t;\varphi_\mu)\right) \tag{5}$$

The background fields $\varphi_\mu \in \mathcal{B}_\mu$ and the functional integral goes over $\mathcal{A}_\mu$. We assumed that the metric does not mix the fields from $\mathcal{A}_\mu$ and $\mathcal{B}_\mu$, but the other terms in the action unavoidably do.

[†] The difference between the strong and weak completeness was recently rediscovered [20] (for an earlier related analisys see [21]) in attempts to test the relation between Polchinski's and the holomorphic RG with the help of the standard formalism of matrix models [14, 16, 17]. Also, to avoid confusion let us emphasize that in the case of D-dimensional field theories the set of functions $\mathrm{Fun}(\mathcal{A})$ includes not just polynomials of ϕ, but also the derivatives of ϕ along various directions in W.

If the set of functions $\mathcal{O}_n$ is *strongly* complete, then we are dealing with a renormalizable family of field theories, and the new action $A_\mu(t)$ belongs to the same moduli space $\mathcal{M}$, i.e. such $t_\mu(t)$ exist that

$$A_\mu(t;\phi) = A(t_\mu, \phi) \tag{6}$$

and

$$Z_\mu(t;\phi_\mu) = Z_\infty(t_\mu, \phi_\mu) \tag{7}$$

Then for two normalization points $\mu < \lambda$ we have:

$$Z_\mu(t;\varphi_\mu) = \int_{\mathcal{A}_\mu/\mathcal{A}_\lambda = \mathcal{B}_\lambda/\mathcal{B}_\mu} Z_\lambda(t;\phi+\varphi_\mu) D\phi \tag{8}$$

This procedure is known as Kadanoff transformation.

3 Polchinski's exact RG equation

A simple way to vary the normalization point μ is provided by the change of metric [2]. It is enough to introduce a μ-dependent family of metrics G_μ, such that $\mathcal{A}_\lambda = \text{supp}\ (G_\mu^{-1})$. This motivates the study of metric dependence of partition function.

The variation of $Z(t)$ with the variation of metric G is:

$$\delta Z(t) = \langle\!\langle \phi \delta G \phi \rangle\!\rangle \tag{9}$$

A Ward identity [2][‡] states that

$$\langle\!\langle \phi\delta G\phi \rangle\!\rangle = - \left\langle\!\!\left\langle \delta G^{-1} \left(\frac{\partial A}{\partial \phi}\frac{\partial A}{\partial \phi} + \frac{\partial^2 A}{\partial \phi^2} + G \right) \right\rangle\!\!\right\rangle \tag{10}$$

For concrete actions of particular models (which do not satisfy completeness requirement), the r.h.s. is an average of a new operator and is not expressible through $Z(t)$. However, for the partition function, built with the help of complete sets of functions the situation is different. For the *strongly* complete set of $\mathcal{O}_n$ one can simply define $\delta t^{(n)}$ from

$$\delta A(\phi) = \delta G^{-1} \left(\frac{\partial A}{\partial \phi}\frac{\partial A}{\partial \phi} + \frac{\partial^2 A}{\partial \phi^2} + G \right) = \sum_n \delta t^{(n)} \mathcal{O}_n(\phi) \tag{11}$$

without any averaging. This is the situation described in terms of RG [2]. Then, eq.(10) can be rewritten as a *linear* differential equation

$$\delta Z(t) = \langle\!\langle \phi\delta G\phi \rangle\!\rangle = \sum_n \delta t^{(n)} \frac{\partial Z(t)}{\partial t^{(n)}} \equiv \delta G \cdot \hat{v}(t) Z(t) \tag{12}$$

In a *weakly* complete case one can not define $\delta t^{(n)}$ from (11), but an analogue of (12) still exists, only it involves a differential operator $\hat{\Delta}$, not obligatory linear in t-derivatives:

$$\delta Z(t) = \langle\!\langle \phi\delta G\phi \rangle\!\rangle = -\delta G \cdot \hat{\Delta}(t) Z(t) \tag{13}$$

or

$$\hat{L}(t) Z(t) \equiv \left(\frac{\partial}{\partial t^{(2)}} + \hat{\Delta}(t) \right) Z(t) = 0 \tag{14}$$

In the case of the matrix models this is exactly (one of the) W- or Virasoro constraints [14, 15, 16]. Of course, there are many more Ward identities for complete partition functions, besides (10). Some of them are actually linear in t-derivatives, but in the weakly complete cases these linear equations do not contain G-derivatives (N-variations in the case of matrix models), and do not help, at least in any straightforward way, to formulate an RG equation.

‡ It follows from the obvious identity

$$0 = \int D\phi \frac{\partial}{\partial \phi} \left\{ \delta G^{-1} \left(\frac{\partial}{\partial \phi} + 2G\phi \right) \exp\left(-\frac{1}{2}\phi G\phi + A(\phi) \right) \right\}$$

4 Quasiclassical RG

As an example of Kadanoff-Polchinski's procedure consider a theory with N copies of every field ϕ and partition function

$$Z_N(t) = \int \prod_{i=1}^{N} D\phi^i \exp\ \hbar^{-1}\left(-\frac{1}{2}\phi^i G_{ij}\phi^j + \sum_n t^{(n)}\mathcal{O}_n(\phi)\right) \tag{15}$$

Let us now add one more field ϕ. Such changing of N can be considered as a result of a change of metric: switching on or off some components of G^{ij}. This change $N \to N+1$ is not infinitesimal, but in the quasiclassical approximation, i.e. in the leading order in $\hbar$ expansions, a result similar to (13) holds:

$$\begin{aligned} Z_{N+1}(t) = \int D\phi e^{-\frac{1}{2\hbar}G\phi^2} \int \prod_{i=1}^{N} D\varphi^i \exp\ \hbar^{-1}\left(-\frac{1}{2\hbar}\varphi^i G_{ij}\varphi^j + \sum_n t^{(n)}\mathcal{O}_n(\phi+\varphi)\right) \times \\ \times \exp\ \hbar^{-1}\left(\phi\sum_n t^{(n)}\frac{\partial \mathcal{O}_n}{\partial\phi}(\varphi) + \frac{1}{2}\phi^2\sum_n t^{(n)}\frac{\partial^2\mathcal{O}_n}{\partial\phi\partial\phi}(\varphi) + O(\phi^3)\right) = \\ = \left\langle\!\!\left\langle \exp\ -\frac{1}{2\hbar\tilde{G}}\left(\sum_{m,n} t^{(m)}t^{(n)}\frac{\partial\mathcal{O}_m}{\partial\phi}\frac{\partial\mathcal{O}_n}{\partial\phi} + O(\hbar)\right)\right\rangle\!\!\right\rangle_N \equiv e^{\hat{D}_N(t)}Z_N(t) \end{aligned} \tag{16}$$

with $\tilde{G} = G + \sum_n t^{(n)}\frac{\partial^2\mathcal{O}_n}{\partial\phi\partial\phi}$. We assumed here that $\phi = \phi^{N+1}$ enters the original Lagrangian in the same way that all the other ϕ's, e.g. all the operators $\mathcal{O}_n$ are $U(N+1)$ symmetric, and took $G_{i,N+1} = 0$ for symplicity. This tree-like formula manifests the relation between Kadanoff-Wilson RG[§] in the quasiclassical approximation and Polchinski's RG eqs. It describes the shift of the classical action provided by one-particle-reducible diagrams.

In matrix models this integration-out procedure, changing the size of $N\times N$ matrices does not lead to such a shift (unless there are $U(1)$ factors in the symmetry group). The difference is that instead of eliminating a single $\phi = \phi^{N+1}$, in the case of matrix model one needs to eliminate the whole vector $\phi^i = \phi^{i,N+1}$ from Hermitean matrix ϕ^{ij}, and the relevant operators $\mathcal{O}_n(\phi)$ (like $Tr\phi^n$) are bilinear in ϕ^i. Therefore there are no contributions of the order $\hbar^{-1}$, and quasiclassical approximation is not a relevant approximation in the case of matrix models. Its proper substitute is the N^{-1} expansion, where a variety of possibilties occurs, depending on the assumed N-dependence of coupling constants.

5 Description in terms of the diffeomorphisms

The adequate description of RG for the *strongly* complete partition functions is in terms of diffeomorphisms [11]:

$$Z(G';t) = Z(G;t'(t)) = e^{\hat{V}(t)}Z(G;t) \tag{17}$$

Here $\hat{V}(t;G',G) = \sum V_n(t)\frac{\partial}{\partial t^{(n)}}$ is a vector field, so that $e^{\hat{V}(t)}$ is a differential operator. Its relation to the shift $t'^{(n)} - t^{(n)} = \tilde{V}_n(t)$ is provided by generic identification [11] of elements from *Diff* $\mathcal{M}$ and *Shift* $\mathcal{M}$ groups acting on the moduli space of coupling constants

$$\exp(\hat{V}) = :\ \exp(\hat{\tilde{V}})\ : \tag{18}$$

The RG equation (17) represents original (bare) partition function as an action of an *operator* on the new (renormalized) partition function. Generically, the vector field $\hat{\tilde{V}}$ decomposes into a $\partial/\partial t^{(0)}$-piece and all the other t-derivatives. The first piece can be considered as generating an additive correction to the effective action $S = \log Z$, while the remaining part of $\hat{V}$ generates shifts of the other couplings. In other words, one can represent (17) in a different form:

$$S(G';t) = S_0(t) + S(G;t'(t)) \tag{19}$$

where S is supposed to depend only on $t^{(n)}$ with $n > 0$, and

$$S_0(t) \equiv t^{(0)} - t'^{(0)}(t) = \log\left(\exp(\hat{V}(t))e^{t^{(0)}}\right) \tag{20}$$

[§] In theories with the power-like scaling laws one often supplements the "integration-out" procedure by afterall rescaling of the world sheet W "back to its original size". Though important for some interpretations of RG equations, this additional procedure is not essential for our purposes.

Relation (19) describes a decomposition of the type suggested in [10]. Moreover, like requested in [10], the two items at the r.h.s. of (19) satisfy the closely related equations. Indeed, a pair of relations,

$$\begin{aligned} \hat{L}(t)Z(G';t) = 0, \\ \hat{L}(t)e^{t^{(0)}} \ = \ const \cdot e^{t^{(0)}} \end{aligned} \tag{21}$$

where the first one is the Ward identity (14) and the second one reflects the fact that the trivial partition function $\exp\left(t^{(0)}\right)$ is usually an eigenstate of $\hat{L}(t)$, turns into:

$$\begin{aligned} \hat{\mathcal{L}}(t)Z(G;t) = 0, \\ \hat{\mathcal{L}}(t)\exp S_0(t) = \ const \cdot \exp S_0(t) \end{aligned} \tag{22}$$

with a new operator

$$\hat{\mathcal{L}}(t) \equiv \exp(\hat{V}(t))\hat{L}(t)\exp(-\hat{V}(t)) \tag{23}$$

The second relation in (22) implies that $S_0(t)$ is non-vanishing when the eigenvalue in (21) is different from zero. If rewritten in terms of effective actions $S(t)$ and in the quasiclassical limit, when $\partial^n Z \to Z(\partial S)^n$, the equations (22) acquire the form of the Hamilton-Jacobi equations. They are quadratic in $S(t)$ if the differential operator in (13) is of the second order in $\partial/\partial t$, what is often the case in some simple models.

Unfortunately, the above reasoning is mixing two different things, which are not obligatory compatible: the RG equation (17), occuring for the *strongly* complete partition functions, and the Laplace-like equation (14), requiring only *weak* completeness. In the strongly complete case, the non-linear (in coupling derivatives) equation, even if occurs, can be always rewritten as a linear equation. In fact, one can easily make a weakly complete model strongly complete, by adding all the newly emerging operators to the action $A(t;\phi)$, then, if the product $\mathcal{O}_m\mathcal{O}_n$ is added with the coefficient $t^{(m,n)}$, we have an identity $\partial^2 Z/\partial t^{(m)}\partial t^{(n)} = \partial Z/\partial t^{(m,n)}$. Alternatively, in the weakly complete case one could try to interpret (12) as a substitute for RG equation, but then in (17) we get $\exp(\hat{V}(t)) = P\exp(\int \delta G\cdot\hat{v}(t))$ substituted by $\exp\hat{D} = P\exp(\int \delta G\cdot\hat{\Delta}(t))$, which is an element of $Dop\mathcal{M}$, but no longer of $Diff\mathcal{M}$.

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References

[1] N.Bogolubov and D.Shirkov, *Introduction to the Theory of Quantum Fields*, Moscow, 1957;
M.Gell-Mann and F.E.Low, Phys.Rev., **95** (1954) 1300;
L.P.Kadanoff, In: *Varenna 1970, Proceedings, Critical Phenomena*, New York 1971, 100-117;
K.G.Wilson and J.Kogut, Phys.Rept., **12C** (1974) 77;
A.Polyakov, *Gauge Fields and Strings*, Harwood Academic Publishers, 1987.

[2] J.Polchinski, Nucl.Phys., **B231** (1984) 269.

[3] For the text-books on RG see:
A.Z.Patashinsky and V.N.Pokrovsky, *Fluctuation Theory of Phase Transitions*, Oxford, England: Pergamon Pr., 1979 (International Series In Natural Philosophy, Vol.98);
S.K.Ma, Rev.Mod.Phys., **45** (1973) 589;
and also recent reviews:
T.R.Morris, Prog.Theor.Phys.Suppl. **131** (1998) 395, hep-th/9802039;
C. Bagnuls and C. Bervillier, hep-th/0002034.

[4] I.Krichever, Comm.Pure Appl.Math., **47** (1994) 437;
B.Dubrovin, Nucl.Phys., **B379** (1992) 627; hep-th/9407018;
H.Itoyama and A.Morozov, Nucl.Phys., **B491** (1997) 529, hep-th/9512161

[5] A.Gorsky, I.Krichever, A.Marshakov, A.Mironov and A.Morozov, Phys.Lett., **B355** (1995) 466, hep-th/9505035;
A.Gorsky, A.Marshakov, A.Mironov and A.Morozov, Nucl.Phys., **B527** (1998) 690, hep-th/9802007.

[6] J.Maldacena, Adv.Theor.Math.Phys., **2** (1998) 231, hep-th/9711200;
S.Gubser, I.Klebanov and A.Polyakov, Phys.Lett., **B428** (1998) 105, hep-th/9802109;
E.Witten, Adv.Theor.Math.Phys., **2** (1998) 253, hep-th/9802150.

[7] A.Gerasimov, *unpublished*;
E.Akhmedov, Phys.Lett., **B442** (1998) 152, hep-th/9806217
E.Alvarez and C.Gómez, Nucl.Phys., **B541** (1999) 441, hep-th/9807226.

[8] D.Z.Freedman, S.S.Gubser, K.Pilch and N.P.Warner, hep-th/9906164;
L.Girardello, M.Petrini, M.Porrati and A.Zafaroni, Nucl.Phys., **B569** (2000) 451, hep-th/9906164;
JHEP, **9812** (1998) 022, hep-th/9810126;
V.Balasubramanian and P.Kraus, Phys.Rev.Lett., **83** (1999) 3605; hep-th/9903190;
K.Skenderis and P.K.Townsend, Phys.Lett., **B468** (1999) 46-51, hep-th/9909070;
O.DeWolfe, D.Z.Freedman, S.S.Gubser and A.Karch, hep-th/9909134.

[9] A.Polyakov, Proceedings of the Les Houches School, 1993;
A.Polyakov, Nucl.Phys.Proc.Suppl., **68** (1998) 1; hep-th/9711002.

[10] J.de Boer, E.Verline and H.Verlinde, hep-th/9912012;
E.Verlinde and H.Verlinde, hep-th/9912018;
E.Verlinde, Class.Quant.Grav. 17 (2000) 1277-1285, hep-th/9912058;
J.Khoury and H.Verlinde, hep-th/0001056.

[11] A.Gerasimov, A.Morozov and K.Selivanov, hep-th/0005053.

[12] A.Morozov, Sov.Physics Uspekhi, **35** (1992) 671.

[13] M.Fukuma, H.Kawai and R.Nakayama, Int.J.Mod.Phys., **A6** (1991) 1385;
R.Dijkgraaf, E.Verlinde and H.Verlinde, Nucl.Phys., **B348** (1991) 435.

[14] A.Mironov and A.Morozov, Phys.Lett., **B252** (1990) 47-52;
J.Ambjorn, J.Jurkiewicz and Yu.Makeenko, Phys.Lett., **251B** (1990) 517-524;
H.Itoyama and Y.Matsuo, Phys.Lett., **255B** (1991) 202.

[15] A.Marshakov, A.Mironov and A.Morozov, Mod.Phys.Lett., **A7** (1992) 1345-1359;
C.Ahn, K.Shigemoto, Phys.Lett., **B285** (1992) 42, hep-th/9112057;
A.Mikhailov, Int.J.Mod.Phys., **A9** (1994) 873, hep-th/9303129.

[16] A.Morozov, Sov.Physics Uspekhi, **37** (1994) 1, hep-th/9303139; hep-th/9502091;
A.Mironov, Int.J.Mod.Phys., **A9** (1994) 4355, hep-th/9312212.

[17] Yu.Makeenko, A.Marshakov, A.Mironov and A.Morozov, Nucl.Phys., **B356** (1991) 574-628.

[18] A.Marshakov, A.Mironov and T.Tomaras, *in preparation*.

[19] A.Connes and D.Kreimer, Commun.Math.Phys., **210** (2000) 249, hep-th/9912092; hep-th/0003188.

[20] M.Li, hep-th/0001193.

[21] R.Brustein and S.P.De Alwis, “Renormalization group equation for string field theory and matrix models,” UTTG-08-91, Presented at PASCOS-91 Int. Symp., Boston, MA Mar 25-30, 1991.